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Counting and Measuring Crosslinguistically
Lecture 2: Counting Measuring and Approximation

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In the previous class I argued:
1. numericals have an interpretation at type <e,t> (numerical modifiers) and an interpretation at type n (name for numbers). Complex numericals are constructed out of lexical powers such as *hundred*, heads at type <n<e,t>>, which combine with numericals at type n to give numerical expressions at type <e,t>.

2. Numericals at type n are used to count as in *one, two, three* etc,
Numericals at type <e,t> are used to express counting evaluations of quantity as in *three cats, four dogs, ten cows.* In this use *three, four* and *two hundred* are **counting predicates**.

3. Numericals at type n are used in expressions like *two kilos. Kilo* is also an expression at type <n, <e,t>>. It combines with *two* at type n to give the predicate expression *two kilos* at type <e,t>. These predicate expressions are **measure predicates.**

In this class I want (i) to contemplate the difference between counting and measuring as it is expressed cross-linguistically and (ii) to consider the relation between the counting/measuring distinction (a distinction in **operations**) and the count/mass distinction (a distinction in **kinds of nouns**).

I shall try and defend the hypothesis that **mass nouns present pluralities in such a way that they can be measured, and count nouns present pluralities to be counted.**

We are going to explore this by looking in particular at quantity evaluations with so-called 'object mass nouns'.

Part I: Comparing object mass nouns crosslinguistically

I. Object mass nouns in English

In typical mass/count languages, numeral modifiers modify count nouns directly. In many languages, with numerals greater than *one* the nominal is marked as plural (1a) vs (1b): Numerical expressions are used with mass nouns (and plural count nouns) as in (1c). expressions of quantity may distinguish between mass nouns and count nouns as in (1d)/

(1) a. three flowers, four books.
b. #three flours, #four muds
c. three kilos of flour, two litres of wine
d. many books, #many mud(s), #not much books, not much mud.

Note that in languages which have the mass/count distinction, i.e. the distinction between (1a) and (1b), this distinction may express itself via different tests.

Object mass nouns have the morphosyntactic distribution of mass nouns, but seem to denote sets of individuable entities. Rothstein (2010) calls them 'naturally atomic mass nouns'.

(2) a. furniture, kitchenware, jewellery, clothing, mail…
b. #one furniture, #three kitchenwares, #two jewelleries….
c. one piece of furniture, three pieces of kitchenware, two pieces of jewellery…

The examples in (2) have been called 'superordinates': they seem to denote sets of entities which can be classified into different kinds of objects.

furniture: chairs, tables, sofas…
jewellery: rings, necklaces, bracelets…
mail: parcels, letters, flyers…
footwear: shoes, boots, sneakers, slippers
kitchenware: cutlery: knives, forks, spoons; crockery: plates, cups, saucers; pots, pans…
clothing: underwear: socks, underpants; coats, dresses,

It seems that in English many object mass nouns are superordinates. One grammatical property which sets them apart: normally mass nouns pluralise with a subkind reading (3). These don't:

(3) a. We had three different wines for dinner: a white, a red Bordeaux, and a dessert wine.

(4) a.#This shop sells many different furnitures: modern, wooden, antique, do-it-yourself.
b. #All the kitchenwares sold by this company are of the highest quality.

Rothstein (2010) and Schwarzschild (2011) observe independently that when predicates of individuals modify object mass nouns, they distribute over the individuals:
In (5a) *big furniture = big pieces of furniture.* Schwarzschild calls predicates like *big* 'stubbornly distributive predicates.'

(5) a. Please carry the big furniture downstairs first. (from Rothstein 2010)
b. The expensive jewellery is on the third floor of the store.

(5) suggests that the individuals in the denotation of object mass nouns are 'conceptually accessible' even though they are not countable

Barner and Snedeker (2005) show the same point experimentally. Under neutral experimental conditions, (6) is evaluated by comparing cardinalities and not volume. In answer to (6), subjects consistently judged 3 small pieces of furniture as more than one big piece, even if the overall volume of the big piece was greater than the overall volume of the three small pieces. Comparison of cardinalities indicates access to atomic structure:

(6) Who has more furniture?

Conclusion: mass nouns may denote sets of individuated entities. In comparison contexts, mass nouns may be compared in terms of the number of their atomic parts.

There are thus two kinds of mass nouns: substance mass nouns e.g. *mud, sand* and object mass nouns. Rothstein (2010) calls object mass nouns *naturally atomic predicates*

Barner & Snedeker (2005) and Bale and Barner (2009) argue for a stronger claim about comparatives:
Cardinal comparisons are not only possible with *furniture* nouns, they are obligatory.
Comparison by cardinality is the only way of answering (6).
If a mass noun can be compared in terms of cardinality, then it must be compared in this way.

Bale & Barner (2009) claim that comparative constructions are the best test of the denotative properties of nouns.
Comparative constructions reveal three kinds of nouns:
(i) **count nouns**: these denote sets of individuals. Comparative operations always compare the cardinalities of sets of individuals. e.g. *boy(s), girl(s),book(s), pen(s)*:

(7) a. There are more boys in your class than in my class.
b. There are more girls in the class than boys.

This is explained if singular count nouns denote sets of well-defined atoms, and plural count nouns denote sets of atoms closed under sum. Comparison involves comparing cardinalities of sets of atoms or plural sums of atoms.

(ii) **object mass nouns**: Since comparative operations always compare cardinalities, these must have the same kind of denotation as plural count nouns. So the mass noun denotes a set closed under sum, generated by a set of well-defined atoms. Examples; *furniture, jewellery*:

(8) a. John has more furniture than Bill = John has more pieces of furniture than Bill.
b. That baby has more footwear than her mother! = She has more shoes than her mother!

Barner & Snedeker and Bale & Barner: these nouns are lexically marked [+individual], and the denotation includes some privileged set of atoms.

(iii) **substance mass nouns**: these denote sets of quantities of non-individuated stuff. There is no privileged set of atoms. Comparative operations compare overall quantities:

(9) a. John has more gold than Bill.
b. There is more mud on this floor than on that floor.

Bale and Barner (2009):
(i) Nouns denoting individuals i.e. count and object mass nouns must be compared via cardinalities, while nouns denoting stuff are compared along continuous dimensions.
(ii) Comparative constructions test whether a mass noun is substance or object.

There is a 4th class of **flexible** nouns like *stone:* These are ambiguous between mass and count:

(10) This garden has more stone/more stones in it than that garden.

This leads to the claim in (iii):

(iii) Flexible nouns are always compared via cardinalities when used as count, but never when used as mass. (11a) compares cardinalities, while (11b) asks for a comparison of overall volume:

(11) a. Which garden has more stones in it?
b. Which garden has more stone in it?

In sum: (i) object mass nouns, or naturally atomic mass nouns have the semantics of count nouns but the syntax of mass nouns. (ii) The mass N in a flexible pair is always a substance mass noun.

Bale and Barner (2009): All mass nouns are 'root nouns'.
Substance mass nouns denote sets of unindividuated entities.
Object mass nouns denote sets of atomic individuals closed under sum.
Count nouns are derived from **substance mass nouns** and denote sets of atomic individuals closed under sum. (A somewhat crude summary, but essentially what they say).
This has a number of unintuitive results: (i) object mass nouns are the only nouns that inherently denote individuals while root nouns like *boy* do not. (ii) despite this they are not countable.

However, most important: the empirical generalization on which this theory is based is not correct. Object mass nouns allow, but do not require cardinality comparisons, and thus must be different from count nouns in a fundamental way. We discuss this in section II.

II. Against the strong version of the claim that object mass noun individuate:
Context-neutral comparison evaluations are naturally cardinal as in Barner & Snedeker 2005, but non-cardinal evaluations are also possible. (12a) contrasts with (12b). See also (12c/d):

(12) a. John has more furniture than Bill, so he will need the larger moving truck.
b. John has more pieces of furniture than Bill, so he will need the larger moving truck.
c. Who has more jewellery to insure?
d. John got sick because he ate more fruit than Mary. She ate two apples and three strawberries. He ate a whole watermelon.

Landman (2011) makes the same point with respect to *most N are P* which compares $N \cap P$ with $N \cap \overline{P}$. When N is count, the truth of *most N are P* depends on the cardinality of :
$N \cap P$ vs. $N \cap \overline{P}$. The examples are based on Landman (2011). (13a) is true, (13b) is false since the cardinality of **farm animals ∩ chickens** is greater than **farm animals ∩ cows**, though the volume is not. When N is a naturally atomic mass noun, the dimension of comparison may vary. (13c) and (13d) are both true and felicitous (13e) is infelicitous.

(13) a. Most farm animals are chickens.
b. Most farm animals are cows.
c. In terms of **number**, most livestock is poultry.
d. In terms of **volume**, most livestock is cattle.
e. #In terms of volume, most farm animals are cattle.

Grimm and Levin (2012) support this data experimentally: they show that judgments of the form *who has more furniture/jewellery?*differ depending on context: in neutral contexts, comparison is usually by cardinality (as in Barner and Snedeker), but in functionally oriented

contexts, the judgement may depend on which set better fulfills the specified function. Fewer items may be considered 'more' if they serve the local purpose better. Gafni & Rothstein (2013) replicates the results for Hebrew.

Crosslinguistic data further supports the position that naturally atomic mass nouns may, but need not, compare via cardinality. We discuss Brazilian Portuguese, Hungarian and Chinese

III. Brazilian Portuguese

Data from Brazilian Portuguese shows that comparison of count nouns is always comparison of cardinalities, but choice of dimension of comparison for object mass nouns is context dependent.

Point 1: Object mass nouns such as *mobília* allow comparison along the dimensions of either cardinality or volume: Out of the blue, (14) compares numbers of pieces of furniture However, the context in (15) forces a context in terms of volume:

(14) João tem **mais mobília** que a Maria.
'John has more furniture than Maria.'

(15) João tem **mais mobília** que a Maria então ele vai precisar de uma caminhote maior.
'John has more furniture than Maria, so he will need a larger moving truck'

Point 2: Brazilian Portuguese allows so-called 'bare singular' uses of count nouns. Pires de Oliveira and Rothstein (2011) (contra Schmitt and Munn 1999 and others): bare singulars are mass nouns. They pattern like mass nouns in two kinds of ways (details in paper):

A. Bare singulars and mass nouns generally have the same distribution and interpretation: e.g. Both have only a generic interpretation in the subject position of stage level predicates, while bare plurals are ambiguous between a generic and an existential interpretation.

(16) a. Bombeiros são disponíveis. (generic OR existential readings)
fireman-PL be.PRS.3PL available-PL.
'Firemen in general are available.' OR 'Some firemen are available.'
b. Bombeiro é disponivel. (ONLY generic reading)
fireman be.PRS.3SG available.
'Firemen in general are available.'
c. Petróleo é disponivel. (ONLY generic reading)
oil be.PRS.3SG available.
'Oil is available.'

(17) a. João gosta de cachorros. (kind OR specimen readings)
João like-PRS.3SG of dog-PL.
'João likes dogs in general.' OR 'João likes some individual dogs.'
b. João gosta de cachorro (kind/*specimen)
João like-PRS.3SG of dog.
'João likes dogs in general.'
c. João gosta de suco. (kind/*specimen)
João like-PRS.3SG of juice.
'João likes juice in general.'

B. Bare singulars and object mass nouns both allow reciprocals and distributive predicates:

(18) a. *Ouro pes-a duas grama-s.
gold weigh-PRS.3SG two gram-PL.
Intended meaning: ‘Pieces of gold weigh two grams’.
b. *Ouro realç-a um ao outro.
gold enhance-PRS.3SG one to.the other.
Intended meaning: ‘Pieces of gold enhance each other.’

(19) a. Criança (nessa idade) pes-a 20 kg.
child (at.this age) weigh-PRS.3SG 20 kg.
‘Children (at this age) weigh 20 kg.’
b. Criança brig-a uma com a outra.
Child fight one with the other
‘Children fight with one another.’

(20) a. Mobília (dessa marca) pes-a 20 kilos.
furniture (this brand) weigh-PRS.3SG 20 kg
'pieces of furniture (of this brand) fit into each other'

b. Mobília (dessa marca) encaix-a uma na outra
furniture this brand) fit one in another
‘Pieces of furniture (of this brand) fit into each other

(Note that there is a real difference between English and Brazilian Portuguese w.r.t (20b):

(21) # Furniture from Ikea fits into each other.

European Portuguese apparently works the same way as Brazilian Portuguese, so the grammaticality of (20b) is not connected to the licensing of bare singulars.)

If bare singulars in Brazilian Portuguese are mass nouns, then **all count nouns in Brazilian Portuguese are flexible nouns**, since all count nouns have a bare singular counterpart. This means that all count nouns in Brazilian Portuguese occur in pairs comparable to *stone/ stones* in English. However, in contrast to English the bare singular *cachorro* ‘dog’, *livro* ‘book’, is interpreted as an object-mass noun, not a substance mass-noun. (Note also that it does not normally have a ‘ground’ reading.) Support for this claim:
(i) in DP and quantificational comparatives the count noun forces cardinality judgements, while the bare singular i.e.object mass noun is context dependent:

(22) a. João tem **mais livros** que a Maria. (cardinal^{ok}, volume*)
‘John has more books than the Maria.’

b. João tem **mais livro** que a Maria. (cardinal^{ok}, volume^{ok})
‘John has more book than the Maria.’

(22a) only compares cardinalities, while (22b) can be used to assert that João has a greater number of individual books than Mary, or that he has a greater volume or weight of book(s) than she has (though fewer books). It does not normally have a ground reading.
Quantifiers show the same pattern: count nouns: (23a) allow only computation in terms of cardinality, mass nouns (23b) allow either a cardinal or a volume reading. (23b) can be answered *two shelves-ful* or *two hundred*!

(23) a. **Quantos livros** ele comprou? (PdeO&R 2011: 52b)
how-many-PL book-PL he buy-PST.PRF.3SG
'How many books did he buy?'
b. **Quanto livro** você comprou?
How book you buy-PST.PRF.3SG
'What quantity of books did you buy?'

(24) a. João tem **muitas caneta(s)/muitos livro(s)**. (cardinal^{ok}, volume*)
'John has many pens/ many books.'
b. João tem **muita caneta/muito livro**. (cardinal^{ok}, volume^{ok})
'John has much pen/ much book'

Note this holds even if plural morphology is dropped from a count noun, as happens in some dialects of BrP, and the count/mass contrast is marked only on the quantifier, as shown in (24a). (25) is a nice additional example from Pires de Oliveira & Rothstein (2011).

(25) a. Essa lata tem **mais minhoca** do que aquela. (PdeO&R 2011: (53))
this can have.PRS.3SG more earthworm-SG of.the than that.
'This can contains a bigger quantity of earthworm than that one.'
b. Não esse tem 10 e aquele tem 12 minhocas.
no, this have.PRS.3SG 10 and that have.PRS.3SG 12 earthworm-PL.
'No, this can has 10, and the other one has 12 earthworms.'
c. Mas esse pesa mais.
But this weigh-PRS.3SG more
'But this one weighs more.'

(ii) measure phases take bare singular i.e. mass complements:

(26) dois quilos de livro/#livros
two kilos of book/books

What this data shows:
i. Quantity evaluations for count nouns are always in terms of cardinality, while quantity evaluations (for object mass nouns) may be in terms of cardinality or along a continuous dimension. This holds also for flexible mass nouns such as *livro/livros* in apparent contrast to English *stone/stones,* where the mass noun resists comparison in terms of cardinality.
ii. Object mass nouns e.g. *livro* have a different denotation from count nouns e.g. *livros*

In the next sections we show that these results are replicated crosslinguistically:

IV Hungarian (Schvarcz 2014, Schvarcz and Rothstein 2015)
Hungarian has two question words: *hány* 'how many' goes with count nouns, while *mennyi* 'how much' goes with mass nouns. Note that count nouns in Hungarian are not marked plural when they occur with numbers. *Mennyi* + substance mass nouns does not allow a cardinal answer, as shown in (27). *Hány* + count nouns only allows a cardinal answer (28):

(27) Mennyi/*hány rizs-(e)-t vettél? Három *(kiló-t.)
How much /*how many rice-OM buy-PAST Three kilo
'How much rice did you buy?' Three kilos.

(28) Hány könyv van a táskádban?
How many book-sg is there the bag+possesive your+suffix in ?
'How many books are there in your bag?'

(28.i) Csak három.
Only three.
'Only three.'

(28.ii). # Három kiló.
Three kilo
'Three kilos.'

Schvarcz and Rothstein (2015): *könyv* is ambiguous between a mass and a count noun. As in Brazilian Portuguese, *könyv* can be modified by *mennyi* or *hány* with the same contrast in interpretation.

(29) Mennyi könyv fér a táská-d-ba?
how much book(SG) fit- 3rd SG the bag-3rd poss-suff. Into
'What quantity of book fits into your bag?'

Crucially appropriate answers can be either a measure expression (30i) or a cardinal number (30.ii):

(30) Mennyi könyv-et tudsz cipelni?
How much book.sg.OM you are able to carry
'What quantity of books are you able to carry?'
(i) Három kiló-t
Three kilo.OM
'Three kilos.'
(ii) Hárm-at
three.OM
'Three'

Assuming that *mennyi* induces a mass usage of the bare noun, volume and cardinal answers are acceptable with the mass counterpart of the count noun in Hungarian too.
Note that count nouns which are not flexible can only be questioned with *hány*:

(31) a. hány bokor?
how many bush
'how many bushes?'

b. #mennyi bokor?
how much bush
'how much bush?'

V Mandarin Chinese
Mandarin: There is no distinction between mass and count nouns. All nouns have mass semantics. Counting individuals requires a sortal classifier as in (32), and Cl_{sortal}+ N has been interpreted as analogous to a count noun predicate in English (Xuping Li, 2011, 2013).

(32) san běn shū
three classifier books.

Comparatives such as *hěn duō* 'much/many' and *tài duō* 'too much/many' can occur both with and without a classifiers. Prediction: when it occurs with a sortal classifier, comparison will be in terms of cardinality, when there is no classifier, the dimension of comparison is open:

(33) nǐ dài tài duō (běn) shū le, xínglǐ huì chāozhòng de
you take too much/many Cl_{volume} book PRF baggage will overweight PRT
'You have taken too many books, your baggage will be overweight.'

This prediction is correct. Both versions of (33) allow for a cardinal evaluation. However, imagine a situation in which the bag contains the two volumes of the Compact Edition of the Oxford English Dictionary, (which weighs in at almost seven kilos). In this case, it is only appropriate to use (33) without the classifier, since only the object mass noun allows comparison on a non-cardinal dimension.

Note that there seems to be a continuum:
Mandarin: all nouns have a mass interpretation (substance or object), no nouns have a count interpretation. No flexible nouns.
Brazilian Portuguese: all nouns have a mass interpretation, many nouns also have a count interpretation. Thus all count nouns are flexible.
Hungarian: Most nouns have a mass interpretation: many nouns have a count interpretation, a few have only a count interpretation. Most count nouns are flexible.
English: Some nouns are mass, some nouns are count, a small number of nouns are flexible.

General conclusion:

1. Count nouns require quantity evaluations via cardinality.
2. Object mass nouns allow quantity evaluations along any contextually relevant dimension.
3. Mass nouns (including object mass nouns) do not allow counting and are not modified by cardinals. (except in Yudja, See Lima 2014, Rothstein 2016).

Part II: Theoretical Implications

We want to answer the following questions:

- What semantic analysis explains why count nouns force cardinal comparisons while object mass nouns allow them?
- Why can we count with count nouns and not with mass nouns?
- **What does it mean to compare two quantities in terms of their cardinality, if you can't count the set members? Put differently, in what way are comparisons in terms of cardinality and counting two different kinds of operations?**

To answer these questions we need (i) a theory of the count/mass distinction which allows us to distinguish between count nouns and object mass nouns. (ii) a theory of counting and measuring which explains the relation or difference between counting and comparing cardinalities.

A. A suitable theory of mass/count nouns. (Rothstein 2010, 2011)

Counting and measuring are two different operations.
Counting is a context dependent operation: we count, in a particular context k, the entities of which in that context are considered atomic entities. *Count nouns* are grammatically countable because they encode the contextual parameter. They denote sets of atoms (or pluralities of atoms) indexed for the context in which they count as atomic. They are of type $\langle e \times k, t \rangle$ and denote sets of entities of type $e \times k$, where k is the relevant index.

Mass nouns denote sets at the simple set type $\langle e,t \rangle$, i.e. sets of entities. The entities in the denotations of mass nouns may or may not be naturally atomic. They cannot be counted, but can be measured.

Rothstein (2010): mass noun and count nouns are derived from root nouns.

$MASS(N_{root})$ is the identity operation on root nouns. N_{mass} is a predicate of type <e,t>. $COUNT(N_{root})$ is an operation which maps N_{root} onto N_k, of type <e×k, t>, denoting the set of (indexed) entities which count as atoms in context k, i.e. a set of ordered pairs <x, k>, where x is an entity in N and k is the context.

Mass nouns are of type <e,t>. They are predicates of individuals. They may or may not be naturally atomic.
Count nouns are of type <e×k, t>. They are predicates of indexed individuals. They are semantically atomic.

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Detailed implementation from Rothstein 2010

1. Nominals are interpreted with respect to a complete atomic Boolean algebra M. Intuitively, M is the mass domain. $\sqcup_M$, the sum operation on M, is the complete Boolean join operation; $\sqsubseteq_M$ is the part of relation on M. This is approximately the model of Chierchia 1998, and we assume with him that the set of atoms A of M is not fully specified, vague. (Nothing rests on this choice of mass domain; we assume it for simplicity.)

2. All nouns are associated with an abstract root noun. The denotation of a root noun, N_{root}, is a subset of M, defined as follows:

For some set of atoms, $A_N \subseteq A$, $N_{root} = {}^*A_N$, where $^*X = \{m \in M: \exists Y \subseteq X: m = \sqcup_M Y\}$

Root nouns are the input to operations deriving N_{mass} and $N_{count.}$ Mass nouns are root nouns, i.e, $MASS(N_{root})$ is the identity function on N_{root}. (Singular) count nouns denote a set of semantic atoms derived from the root noun relative to a particular context.

Definition 1:
$MASS(N_{root}) = N_{root}$

3. Count nouns presuppose a context dependent choice as to what counts as one entity. This choice is encoded in the notion of (counting) context k, which intuitively collects together the entities which count as atoms in k.

Definition 2:
A **context** k is a set of objects from M, $k \subseteq M$; K is the set of all contexts.
The set of count atoms determined by context k is the set $A_k = \{<e,k>: e \in k\}$

4. Singular count nouns are derived from root nouns by a count operation $COUNT_k$ which applies to the root noun N_{root} and picks out the set of ordered pairs
$\{<e, k>: e \in N \cap k\}$, i.e. the set of entities in N_{root} which count as one in context k.

Definition 3:
For any $X \subseteq M$: $COUNT_k(X) = \{<e, k>: e \in X \cap k\}$
The interpretation of a count noun N_{count} in context k is: $COUNT_k(N_{root})$.

We use N_k as short for $COUNT_k(N_{root})$, the interpretation of a count noun in context k.

Singular count nouns denote sets of **semantic i.e. indexed atoms.**

Examples: $[\![\text{stone}_{\text{mass}}]\!] = \text{MASS}(\text{STONE}_{\text{root}}) = \text{STONE}_{\text{root}}$
$[\![\text{stone}_{\text{count}}]\!] = \text{COUNT}_k(\text{STONE}_{\text{root}}) = \{<e, k>: e \in \text{STONE}_{\text{root}} \cap k\}$

So $\textit{stone}_{\text{mass}}$ denotes a set of quantities of stone, while $\textit{stone}_{\text{count}}$ denotes a set of type $< e\times k, t>$, namely:
$\{<e, k>: e \in \text{STONE}_{\text{root}} \cap k\}$ i.e. the set of indexed entities which count as one in context k.

5. Plural count nouns are derived by applying the standard plural operation * to the first projection of N_k. $*(N_k)$, the plural of the set of ordered pairs denoted by N_k, is the set of ordered pairs whose first projection, $\pi_1(N_k)$, is the plural set derived from the first projection of N_k, and whose second projection, $\pi_2(N_k)$, is the (same) value k.

Definition 4:
Assume: $\pi_1(N_k) = \{e: <e ,k> \in N_k\}$
$\pi_2(N_k) = k$
In default context k: $PL(N_{\text{count}}) = *N_k = \{<e,k>: e \in *\pi_1(N_k$

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B. A suitable theory of counting vs measuring:

Counting and measuring are two different operations:
Counting: putting entities in one-to-one correspondence with the natural numbers.
Measuring: is assigning to a quantity an overall value on a dimensional scale.

Counting: is putting atomic entities in one-to-one correspondence with the natural numbers. Counting gives a value to a plurality α by assigning numbers from the sequence of natural numbers in order to the atomic parts of α. α has value n if the start and final numbers are n places apart.
Counting is context dependent in two ways:
(i) we always count instances of a specific N. This is why counting *N and N* is difficult:

(34) a. How many cups and saucers are there?
b How many cats and dogs do you have?

(ii) A cardinality property is also dependent on the particular choice of what counts as *one item* of N contextually, and thus dependent on choice of context k. So cardinal predicates are of type $<<e\times k, t> <e\times k,t>>$, they are functions from count predicates into count predicates. *Three* has the denotation in (34) (x an entity of type $e\times k$):

(35) $\lambda x.\ |\pi_1(x)|_{\pi_{2(x)}} = 3 \quad \leftrightarrow \quad \lambda x.\ \big|\ \{y: y \sqsubseteq_{\text{ATOM}} x\}\ \big| = 3$

Three denotes the set of pluralities of type $e\times k$, where e has three parts which count as atomic Ns in k.

Counting focusses on the atomic structure of the plural quantity. If a plurality has a value n, we know something about its internal structure, namely it has n atomic parts, in context k.
Counting is always relative to context k, and cardinals are functions at type $<<e\times k, t> <e\times k,t>>$. Thus mass nouns at type <e,t> cannot be counting.

Measuring is assigning a quantity a value on a scale:

A ***scale*** $S_{M,U}$ is a partial order $S_{M,U}$ = <**N,** $\geq_{M,U}$.$\text{MEASURE}_{M,U}$ > where:

M is a dimension (e.g. volume, weight).

U is the unit of measurement in the relevant dimension, in terms of which the scale is calibrated (e.g. *litre, kilo,*)

N is the real numbers, or the positive real numbers, or a subset of the real numbers, depending on the nature of the measure and the fine-grained of the measurements.

$\text{MEASURE}_{M,U}$ is a function from objects to values in N.

Because the range of values is the set of the real numbers, measuring is inherently **continuous** (see Landman 2016)). We make it non-continuous by choosing a subset of the real numbers as our contextually relevant range of values. Our choice of values determines the **granularity** of the scale. (Solt 2015)

A **measure head** such as *litre* has the denotation in (36a):

A ***measure predicate*** expresses the property of having a particular measure value, (36b):

(36) a. ⟦litre⟧ = $\lambda n \lambda x.\ \text{MEAS}_{\text{VOLUME, LITRE}}(x) = n$

b. ⟦3 litres⟧ = $\lambda x.\ \text{MEAS}_{\text{VOLUME, LITRE}}(x) = 3$

c. ⟦3 litres wine⟧ $\lambda x.\ \text{WINE}(x) \wedge \text{MEAS}_{\text{VOLUME, LITRE}}(x) = 3$

Measuring focusses on the properties of the quantity as a whole. If a quantity x measures three litres, we know nothing about its internal structure. We only know that any way we break x into two non-overlapping parts a and b, the following will hold:

If $a = b \sqcup c$ and $\text{MEASURE}_{\text{VOLUME,LITRE,}}(a) = 3$, then
$\text{MEASURE}_{\text{VOLUME,LITRE,}}(b) = 3 - \text{MEASURE}_{\text{VOLUME,LITRE,}}(c)$

Comparison: *Who has more x?* compares the values assigned to two sets by either the counting or the measure function. *More* does not specify which operation to use. *How much/how many* does specify this (37). See also (38). In other constructions/languages, the operation is not specified lexically (39). In these cases, the choice of operation depends on whether the N is mass or count.

(37) a. How much#many wine did you drink?
b. How many/#much bottles of wine did you drink?

(38) a. John drank fewer/#less bottles of wine than Mary
b. John drank less/#fewer wine than Mary

(39) a. kama bakbukey yayin šatit?
KAMA bottles of wine you-drank?
"How many bottles of wine did you drink?
b. kama yayin šatit?
KAMA wine you-drank?
"How much wine did you drink?"
c. Who listened to more music/more pieces of music?

C. How does the N determine whether counting or measuring is used?
When the noun is count, the atomic structure encoded in the denotation makes the atoms salient and comparison must be via the counting function. *Who has more books/pieces of furniture?* requires a comparison of the value of the counting operation applied to each set.

(40) John has five folding chairs, Mary has a double bed , a grand piano and a piano stool.
Who has more pieces of furniture?

Note that you don't always count in order to answer *Who has more books? How many people are there in the room?* Sometimes you count, sometimes you estimate or calculate. But the answer always involves a comparison of cardinalities, and the correct answer is always in terms of which counting value is higher in the sequence of natural numbers.

When the noun is mass, counting is impossible, and comparison is comparison via the measure function along a contextually relevant dimension.

x is more than y in terms of cardinality: $|x| > |y|$

x is more than y in terms of a measure function: $\text{MEASURE}_{M,U}(x) > \text{MEASURE}_{M,U}(y)$

C. Comparing object mass nouns in terms of cardinalities – how do we do this?

How do we compare mass nouns in terms of cardinalities if we can't count the atoms and can't compare the values of the counting operation? How can we measure cardinalities?
It has been shown experimentally that humans can compare cardinal values without counting. using what is called the Approximate Number System (Hyde 2011, Hyde and Spelke 2009) Preverbal infants and animals can compare cardinalities successfully. (Feigenson, Dehaene, & Spelke, 2004) This is evidence for non-linguistic comparison of quantity-of-individual values on a scale. I argue this is a kind of measuring. This is the basis for expressing comparisons of quantities of objects with object mass nouns.

Comparison of cardinalities without counting is, like other measure comparisons, the comparison of values assigned to quantities on a scale. We can compare values on the cardinal scale without counting. More precisely, we can compare position on a scale without explicit counting. X is more than Y if the value assigned to X is higher on the scale than the value assigned to Y. This kind of comparison can be done via **estimation** or **approximation**.

To do this we need a cardinality scale.
We construct from a set of numbers a scale:

A cardinality scale is an order $S_{\mathbf{CARD},|\,|^{\mathbf{k}}} = \langle \mathbf{N}, \geq_{N.} |\,|^{\mathbf{k}} \rangle$
where CARD stands for cardinality,
k is the context that determines the set of atoms,
N is the set of natural numbers,
and $|\,|^{\mathbf{k}}$ is the function that maps x onto the values in N depending on the cardinality of the set of minimal parts of x relative to k i.e. $|\{y: y \sqsubseteq x\} \cap k|$

The dimension of the scale is arbitrary, since it does not give a value on a particular dimension. (Any sequence can be used to model the natural numbers. See Wiese 2003 for discussion.) N is the set of natural numbers (i.e. the values are not continuous). The units are context determined (via k). A value on a cardinality scale thus gives you a number of units, but does not specify a dimension and thus the information is purely quantitative.

Who has more furniture? = Which set is assigned a higher value on the cardinality scale?

x is more furniture than y:
For x ∈ *furniture* ∧ y ∈ *furniture:* $\text{MEASURE}_{\textbf{CARD},||}{}^{\textbf{k}}(x) > \text{MEASURE}_{\textbf{CARD},||}{}^{\textbf{k}}(y)$

x is more pieces of furniture than y:
For x ∈ *pieces of furniture* ∧ y ∈ *pieces of furniture:* $|\mathbf{x}| > |\mathbf{y}|$

Measuring is assigning a quantity a value on a scale, while counting involves individuating the atomic parts.

Measuring cardinalities is more likely to be approximative than precise, since we give an overall value in terms of atomic parts without actually individuating the atoms. And indeed we find in Brazilian Portuguese that the count questions *quantos livros* is more likely to expect a precise answer, while the mass noun *quanto N* is a bit more appropriate with a round approximate answer:

(41) a. Quanto livro tem na sua biblioteca? #1033/ Por volta de 1000
'How much book are in your library.' #1033/Around 1000
b. Quantos livros tem na sua biblioteca? 1033/ Por volta de 1000
'How many books are in your library' 1033/Around 1000

Schvarcz (pc) reports the same contrasts in Hungarian.

D. Some conclusions

Why do count nouns force cardinal comparisons while object mass nouns merely allow them?
Count nouns make semantic atomic structure salient and thus comparison is via the counting operation, in terms of numbers of atoms. Mass nouns are compared in terms of measure operations. For naturally atomic mass nouns, the cardinal scale is one relevant dimension, but there may also be others.

Why can we count with count nouns and not with mass nouns?
Counting is sensitive to semantic atomicity, the grammatical encoding of atomic structure.

In what way are cardinality comparisons with mass nouns and counting different?
Counting entities is possible when the atomic structure of N is grammatically encoded. Measuring on a cardinality scale involve assigning to a sum an overall value the scale. Obviously two such values can be compared with respect to their position on the scale, even if we don't have the numerical content of these values.

Is all approximation measuring? No, we have approximate counting too. As in "there are about thirty people in the room."
This can be the result of:
(i) indirect counting. "I have thirty homeworks, so there are at least thirty people in the class"
(ii) approximate calculation "There are five rows of seven chairs, and few empty spaces. That must be around thirty people."
(iii) inaccurate counting. "I counted thirty people, but I may have missed someone (or counted someone twice".

In sum:
Count nouns make cardinality the only relevant parameter for quantity evaluation.
Mass nouns allow quantity evaluation on any relevant scale including a cardinality scale.
Cardinality judgments with mass nouns involve grammatical measuring and not counting.

E. A remaining issue: Brazilian Portuguese flexible nouns differ from English flexible nouns.

English 'flexible' nouns: e.g. *stone/stones.* Count readings require cardinal evaluations, mass nouns require non-cardinal evaluations. (Barner and Snedeker 2005, Barner and Bale 2009).
Brazilian Portuguese: e.g. *livro/livros.* Count nouns require quantity evaluation and comparison in terms of cardinality. Mass nouns are neutral and allow quantity evaluation along any contextually relevant dimension, including sizes of cardinalities.

Why are English and Brazilian Portugues flexible nouns different? A pragmatic explanation:
In Brazilian Portuguese: All count nouns are flexible. Mass is grammatically default. Some mass nouns have count counterparts. Count syntax is non-default and forces an interpretation in terms of semantic atoms. Since mass is default, it does not privilege any particular reading.
In English: Flexible nouns are comparatively rare, and neither mass nor count syntax is default. Thus, both privilege a particular interpretation. Choice of count privileges a cardinal interpretation. Choice of mass privileges a non-cardinal interpretation.

F. Absolutely the final section: evidence that estimation or approximation can be a form of measuring

Mandarin: In Mandarin counting and measuring is via classifiers:

(42) san ge xuesheng
three Cl student
'three students'

[Num Cl *de* N] phrases are interpreted as measures and not counting (Cheng & Sybesma 1998).

(43) a. san bang (de) rou
three Cl-pound DE meat
'three pounds of meat'

b. san ge (*de) xuesheng
three Cl (*DE) student
'three students'

It is ruled out with individuating classifiers (44b) and counting uses of container classfiers: (Li 2011, Li and Rothstein 2012).

(44) a. fuwusheng kai le san ping (*de) jiu, yi zhuo yi ping.

waiter open PFV three Cl-bottle DE wine one table one Cl-bottle
'The waiter opened three bottles of wine, one bottle for one table. '

b. ta he le san ping (de) jiu.
he drink PFV three Cl-bottle DE wine
'He drank three bottles of wine. '

However, *de* phrases are possible with high round numbers (Tang 2005, Hsieh 2008) and approximative contexts. Examples from Li and Rothstein (2012), taken from PKU Corpus.

(45) a. mingtian de huodong xuyao yi bai zhang de fangzuozi.
tomorrow mod activity need one hundred Cl-piece DE square table
'Tomorrow's activity needs one hundred square tables. '

b. nabian bian zhong le qi ba ke, shi lai ke de juzi shu.
there then plant PFV seven eight Cl, ten around Cl DE mandarin tree
'On that side were planted seven or eight, or around ten mandarin trees. '

The *de* shows that these constructions involve measuring.
Hypothesis: In this construction we build a non-defective cardinality scale: <DIM, UNIT, N>. The classifier gives the content of the unit. This allows us to measure the overall value of the quantity in terms of the unit given. (45) measures the quantity of tables to be equivalent to 100 *zhang*- units. Crucially these examples are approximations or estimations. Exact figures are infelicitous because they can be acquired only by counting and not by measuring.

(46) a. #women xuyao yi- bai- ling- ba zhang de fangzuozi.
we need one-hundred-zero-eight Cl DE square-table
'We need one hundred and eight square tables. '

b. #yinian zhongzhi-le yi- bai- sanshi-qi ke de shumu.
one-year plant-PFV one-hundred-thirty-seven Cl DE tree
'(They) planted one hundred and thirty seven trees a year.'

X. Selected References:

Bale, A. & D. Barner, 2009. The interpretation of functional heads: Using comparatives to explore the mass/count distinction. *Journal of Semantics*, 26, 217-252

Barner, D., & J. Snedeker, 2005. Quantity judgements and individuation: Evidence that mass nouns count. *Cognition, 97*, pp. 41-66.

Czirmás, Aniko & Éva Dékány (2014). Hungarian is a classifier language. In *Proceedings of Word Classes 2010.* John Benjamins

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number.*Trends in cognitive sciences*, *8*(7), 307-314.

Gafni, Chen 2013 The semantics of mass and count nouns in Hebrew. Term paper. Bar-Ilan University.

Hyde, D. C. 2011. Two systems of non-symbolic numerical cognition.*Frontiers in Human Neuroscience*, *5*.

Hyde,D.C.,and Spelke,E.S.(2009).All numbers are not equal: an electro- physiological investigation of large and small number representations. *Journal of Cognitive Neurosciencei*. 21, 1039–1053.

Khrizman Keren, and Susan Rothstein 2013. *Approximative inversion in Russian as a Measure Construction.* Paper presented at FDSL 10. Leipzig December 2013.

Krifka, Manfred 1989. Nominal Reference, Temporal Constitution and Quantification in Event Semantics. In R. Bartsch, J. van Benthem, P. von Emde Boas (eds.), *Semantics and Contextual Expression*, Dordrecht: Foris Publication.

Landman, Fred, 2010. Count nouns – mass nouns – neat nouns – mess nouns. *The Baltic International Yearbook of Cognition, Logic and Communication.* Vol.6 DOI: 10.4148/biyclc.v6i0.1579

Landman, Fred in press. Iceberg semantics for count nouns and mass nouns: the evidence from portions. In: S. Rothstein and J. Šķilters (eds.), *The Baltic International Yearbook of Cognition, Logic and Communication Volume 11: Number: Cognitive Semantics and Crosslinguistic Approaches.* New Prairie Press, Kansas. (online)

Pires de Oliveira R, and Susan Rothstein. 2011 Bare nouns in are mass in Brazilian Portuguese. *Lingua* doi:10.1016/j.lingua.2011.09.004

Rothstein, Susan 2010. Counting and the mass-count distinction. *Journal of Semantics.* 27. doi:10.1093/jos/ffq007

Rothstein, Susan 2011. Counting, measuring and the semantics of classifiers. *The Baltic International Yearbook of Cognition, Logic and Communication.* Volume 6 http://thebalticyearbook.org/journals/baltic/issue/current

Rothstein, Susan 2016 *Semantics for Counting and Measuring*. Forthcoming Cambridge University Press.

Schvarcz, Brigitta. (2014) Issues in numbers, counting and measuring in Hungarian. MA thesis Bar-Ilan University

Schvarcz, Brigitta R., and **Susan Rothstein,** (to appear). Hungarian classifier constructions and the mass/count distinction. In: A.Liptak and H. van der Hulst (eds.), *Approaches to Hungarian* 15. Amsterdam: John Benjamins.

Schwarzschild, R. (2011) Stubborn distributivity, multiparticipant nouns and the count/mass distinction. In S. Lima, K. Mullin and B. Smith (eds.) *Proceedings of the 39th NELS Meeting,* Amherst MA: GSLA, Volume 2 661-678.

Solt, Stephanie 2015. Vagueness and Imprecision: Empirical Foundations. *Annual Review of Linguistics*. 107-127.

Wiese, Heike 2003. *Numbers, Language, and the Human Mind*. Cambridge University Press.